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Mathematical models to estimate forage production in southeastern Coahuila, Mexico

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ABSTRACT

Objective: To calibrate two non-linear models, in three intermediate triple hybrids, by theoretically comparing the accumulation of dry matter in relation to the days after sowing (das).

Methodology: The cuts were made every 14 days, from 30 to 170 days after sowing, and were adjusted to the Logistic and Richards models. The experimental design was a randomized block, with three replications.

Results: The models explained most (83%) of the total variability of dry matter (DM) yield in maize observed in the field. The best fit model was the Logistic model (cultivar AN447) and the Richards model (cultivar A7573), both with R^2=0.98. The maximum yield simulated with the Richards model was observed in AN447 (22,616 kg DM ha^{-1}) and the lowest in AN388 (10,970 kg DM ha^{-1}).

Limitations/Implications: The results can only be applied to the study case, as a consequence of the limitations imposed by the variety, climate, and soil conditions. Therefore, no general explanation can be developed and the conclusions should be treated with caution.

Conclusion: The Logistic model enables a more precise simulation of the dry matter yield in maize, using the days after sowing as an independent variable.

Keywords: *Zea mays* L., mathematical modeling, and goodness of fit.

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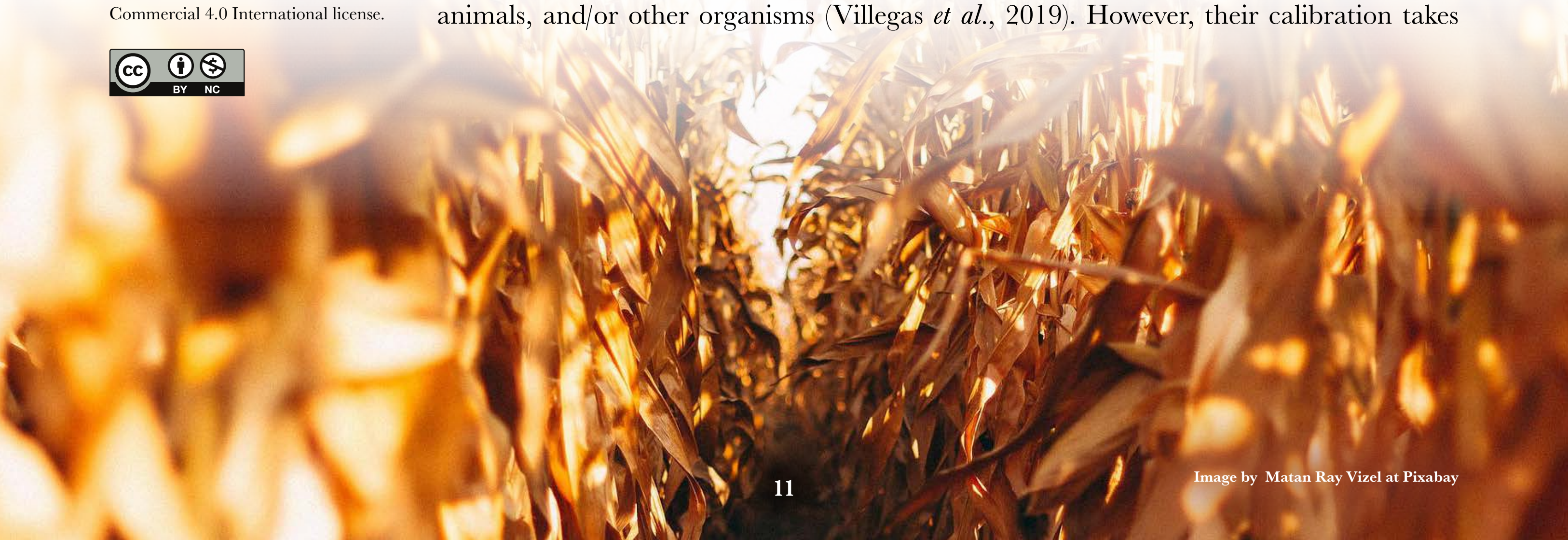
INTRODUCTION

Maize (*Zea mays* L.) is considered the most cultivated cereal worldwide (FAO, 2019). However, climatic variability and management have generated uncertainty and instability in the productivity of this crop; therefore, calibrated simulation models have become a viable tool to study its behavior (Flores *et al.*, 2013). The Logistic (Nelder, 1961) and Richards (1959) models are the most commonly used to describe the growth of plants, animals, and/or other organisms (Villegas *et al.*, 2019). However, their calibration takes



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into consideration the physiological aspect of the plant and its productivity; therefore, as more information is generated, the model fits with greater accuracy (Sinclair and Seligman, 1996). Another important aspect is the validation of the model, carried out comparing the data obtained in the field versus the data obtained by the model and its subsequent application in regions with similar characteristics to the region where it was validated (Boons *et al.*, 1993). In forage species, the models have a good fit (experimental data *vs.* simulated data); such is the case of *Panicum maximun* Cv. Mombaza and *Pennisetum purpureum* Cv. Cuba CT-115 (Rodríguez *et al.*, 2007; Thornley and France, 2007). Many mathematical models simulate maize's development and growth (Lizaso *et al.*, 2011). However, some of these models are too sophisticated and their calibration, validation, and implementation require a large number of parameters and specific skills. Consequently, they are still exclusively for scientific use (Sau *et al.*, 2012) and farmers have a difficult time adopting them (Heng *et al.*, 2009). Therefore, the objective was to calibrate two non-linear models, in three intermediate triple hybrids, by theoretically comparing the accumulation of dry matter in relation to the days after sowing.

MATERIALS AND METHODS

Study area: The work was established in the "El Bajío" experimental area of the Universidad Autónoma Agraria Antonio Narro (UAAAN), in Saltillo, Coahuila, Mexico (25° 23' 12.7" N, and 101° 00' 9.8" W, at 1783 m.a.s.l.). The climate is temperate semi-dry, with temperatures that surpass 18 °C and fall below 0 °C. The average annual accumulated precipitation is 340 mm (García, 2004). Figure 1 shows the weather conditions during the study (Red Universitaria de Observatorios Atmosféricos, UAAAN).

Three intermediate triple maize hybrids (AN447, AN388, and A7573) from the Mexican Corn Institute were used; they were established on April 8th, 2017. The materials were distributed in a randomized block design, with three replications. Each replication consisted of five 9-m long furrows, generating 18 experimental units. The land was plowed, twice harrowed, and furrowed at 0.8 m. Plant density was 86,000 plants ha^{-1}. Irrigation

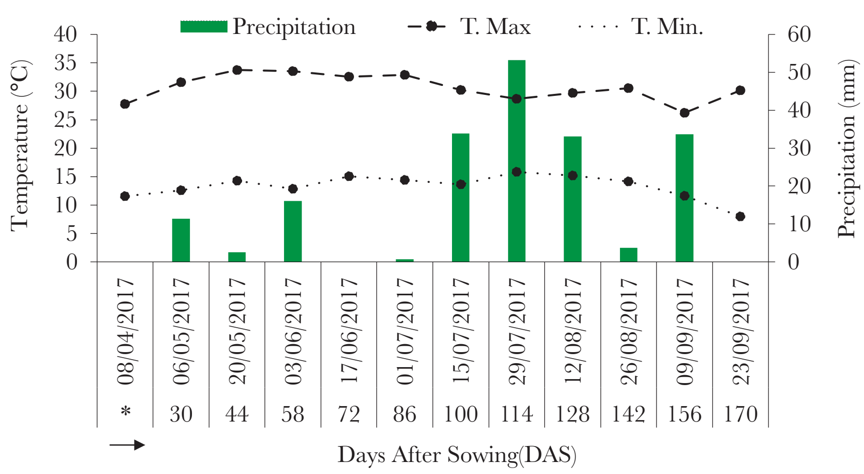

Figure 1. Average maximum and minimum temperatures, and fortnightly accumulated precipitation during the study period (April 8th to October 21st, 2017). *Sowing.

was applied once a week at field capacity, by drip irrigation, using drip tape (wall thickness 6 mil), with a 15-cm distance between drippers. The predominant soil has a sandy-clay-loam texture, with 62, 10, and 20% sand, silt, and clay, respectively; it has 3.02% organic matter and 1.25 g cm^3 apparent density, determined at the beginning of the experiment. Cuts were made every fourteen days from May 6^{th} (30 das) to September 23^{rd}, 2017 (170 days). Samples were taken from three plants and brought to constant dry weight, in a POM-246F forced air stove, at 55 °C during 72 h, in order to determine the accumulation of dry matter. Relationships were established between cutting age after sowing and biomass yield. To estimate the dynamics of growth and dry matter production, two non-linear models were considered: Logistics and Richards (Table 1). The regressions were adjusted with the Statistical Package for Social Sciences software (SPSS, 2011), in which the significance of the correlation coefficients was calculated ($p<0.05$).

RESULTS AND DISCUSSION

Goodness of fit of the Logistic and Richards models

The best model of the AN447 and AN388 hybrids was obtained with the Logistic model; however, the A7573 presented a better fit with the Richards model. The R^2 was 0.98 (AN447), 0.96 (AN388), and 0.98 (A7573) (Table 1). This behavior tends to form a sigmoid curve, similar to the growth dynamics of forage plants (Thornley and France 2007; Martínez *et al.*, 2010). In the case of maize, the field values obtained with other models (*e.g.*, MCWLA-Maize) differed by 2% from those estimated by the model with R^2=0.70, showing good prediction capacity (Tagarakis and Ketterings, 2017).

Estimation of variety AN447 dry matter

The precision with which the models explain the DM production in the AN447 hybrid (depending on the age of the plant) was higher in the Logistic model (R^2=0.98), than in Richards model (R^2=0.84), indicating a lower variability of the Logistic model *vs*. Richards (Figure 2). Although the Logistic and Richards models had predicted 22,536 kg DM ha^{-1} and 22,616 kg DM ha^{-1} DM accumulation, respectively, the actual highest accumulation

Table 1. Non-linear models used to estimate the dry matter yield (W_t) in three maize hybrids with irrigation, as the age of the plant increases in southeastern Coahuila, Mexico.

Model	Funtional form	Differential form
Logistic	$W_t=\dfrac{W_0W_f}{W_0+\left(W_f-W_0\right)e^{-\mu t}}$	$\dfrac{dW_t}{dt}=\mu W\left(1-\dfrac{W}{W_f}\right)$
Richards	$W_t=\dfrac{W_0W_f}{\left[W_0^n+\left(W_f^n-W_0^n\right)e^{-kt}\right]^{1/n}}$	$\dfrac{dW_t}{dt}=\dfrac{k}{n}W\left(1-\left(\dfrac{W}{W_f}\right)^n\right)$

Where: W_0=Initial dry weight, W_f=Maximum or final dry weight, μ=Relative or specific growth rate, e=Base of natural logarithms, t=Time. For the case of the Richards model: n=Form parameter, and k=Constant parameter. The values of μ and k were obtained by simple regression. The values of n were considered according to Thornley, J. H. M. and France, J. (2007).

Table 2. Goodness of fit models for the dry matter yield of three maize hybrids, developed from 30 to 170 days after sowing, at 14-day intervals in southeastern Coahuila, Mexico.

	Cultivars					
	AN447	**AN388**	**A7573**	**AN447**	**AN388**	**A7573**
	Logistics model			**Richards model**		
R^2	0.98	0.96	0.94	0.84	0.83	0.98

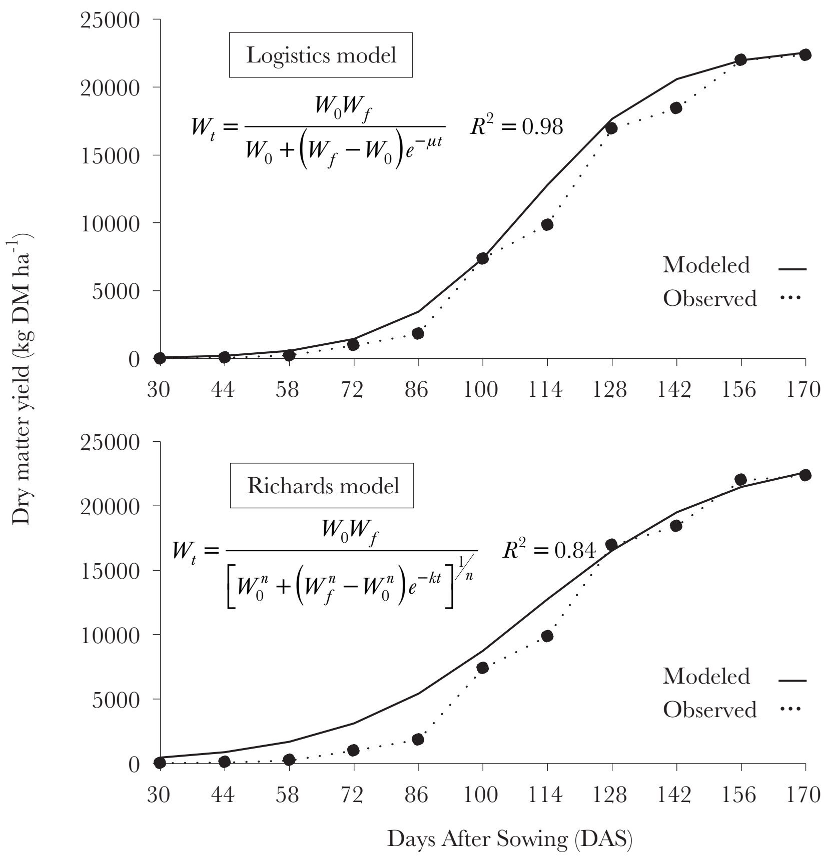

Figure 2. Dry matter yield (kg DM ha^{-1}) of AN447 maize hybrid obtained in the field and simulated with the Logistic and Richards models. Production cycle: April 8th to September 23rd, 2017. Location: southeastern Coahuila, Mexico.

of DM observed in the field at 170 das was 22,361 kg DM ha^{-1}. Both models overestimated the performance by 7% (Logistic) and 12% (Richards); however, the behavior registers a turning point from 156 das. This suggests that the optimal harvest time for this hybrid is after the said date, as reported by Machado *et al.* (1983). These data match the findings of González *et al.* (2014), Diaz *et al.* (2018), and Tornés (2016), who estimated average yields of 20,680 kg DM ha^{-1} at 125 das, obtained with the FAO AquaCrop simulation model. In this regard, determining the optimal harvest time based on simulated values allows a maximum handling of the study materials (Castro *et al.*, 2017), taking into account their growth, which can vary according to management, species, cultivar, and edaphoclimatic conditions (Torres *et al.*, 2012).

Estimation of variety AN388 dry matter

The values estimated with the Logistic model are closer to the observed values than those obtained with the Richards model, since the adjustment generated a correlation of 0.96 and 0.83, respectively. Throughout the study, the variation of DM accumulation in the observed values ranged from 12 to 13,261 kg ha^{-1}. In the Logistic model, the simulated yields varied between 190 and 12,070 kg DM ha^{-1}, while the Richards model recorded variations between 287 and 10,970 kg DM ha^{-1}. Both models overestimated the yield by 2%, which suggests good agreement and little variability. Días and Villalobos (2018) validated the FAO AquaCrop model and reported yields of 28,600 kg DM ha^{-1}. The average difference with the simulations was 670 kg DM ha^{-1} (6% underestimation). This result indicates good biomass predictions (R^2=0.96) and a strong significant relationship between simulated and observed values. Conde *et al.* (2004) calibrated the DSSAT_CERES model for forage maize and overestimated the actual yield by 5-12% for a temperate region of Mexico, which suggests a good fit. In contrast, Arce *et al.* (2017), under similar conditions, estimated a 49.3% decrease in forage maize yield (1-1.7 t ha^{-1}) using AquaCrop, which shows a high degree of variability.

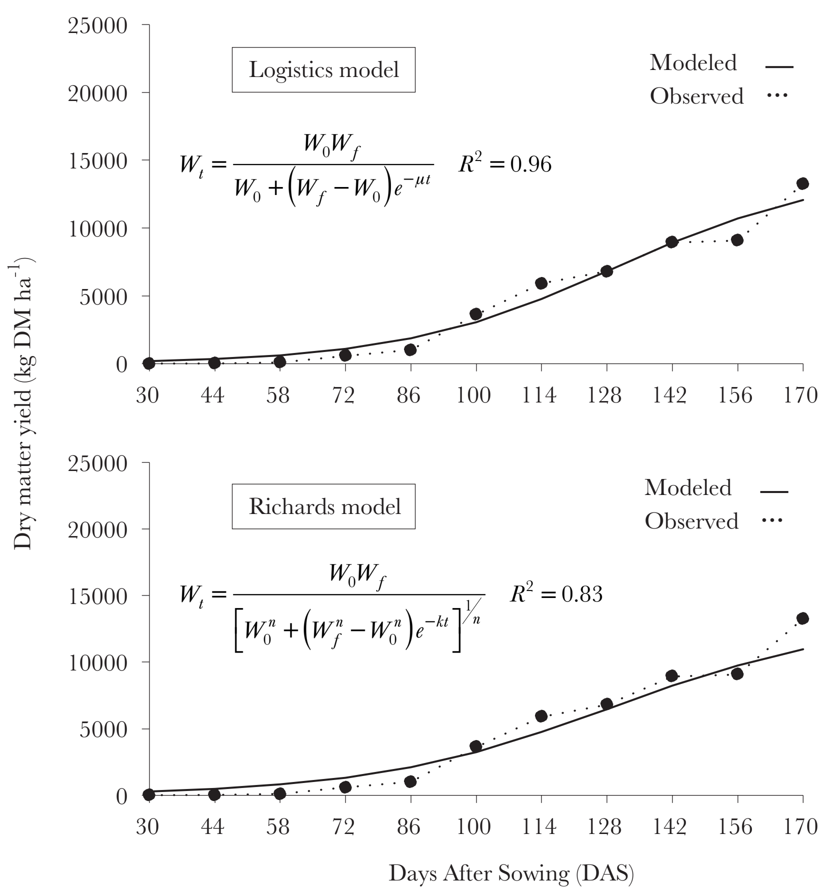

Figure 3. Dry matter yield (kg MS ha^{-1}) of AN388 maize hybrid obtained in the field and simulated with the Logistic and Richards models. Production cycle: April 8th to September 23th, 2017. Location: southeastern Coahuila, Mexico.

Estimation of variety A7573 dry matter

The inflection point of cultivar A7573 took place at 156 das and the maximum value (18,603 kg DM ha^{-1}) was recorded at 170 das (Figure 4). On average, the Logistic model and the Richards model overestimations reached 1 and 2%, respectively. Therefore, both models adequately explain the growth of cultivar A7573, as a result of the good fit of both models —which have correlation coefficients of 0.94 (Logistics) and 0.98 (Richards). The range of dry matter yield in the observed values, from the beginning to the end of the experiment, ranged from 20 to 18,603 kg ha^{-1}. There was a 1 and 2% difference between the values obtained with the Logistic and Richards models regarding the observed values from 30 to 114 das. Subsequently, both models overestimated the yield by approximately 18%. Nouna *et al.* (2000) recorded average differences of 7.27 and 11.71% between the observed and simulated values; differences up to 10% were considered as an acceptable data.

Overall, the highest yields observed were registered by the cultivar AN447, with 9,098 kg DM ha^{-1} average values; meanwhile, the simulated Logistic model recorded 9,880 kg DM ha^{-1} and the Richards model registered 10,291 kg DM ha^{-1} ($p>0.05$); the

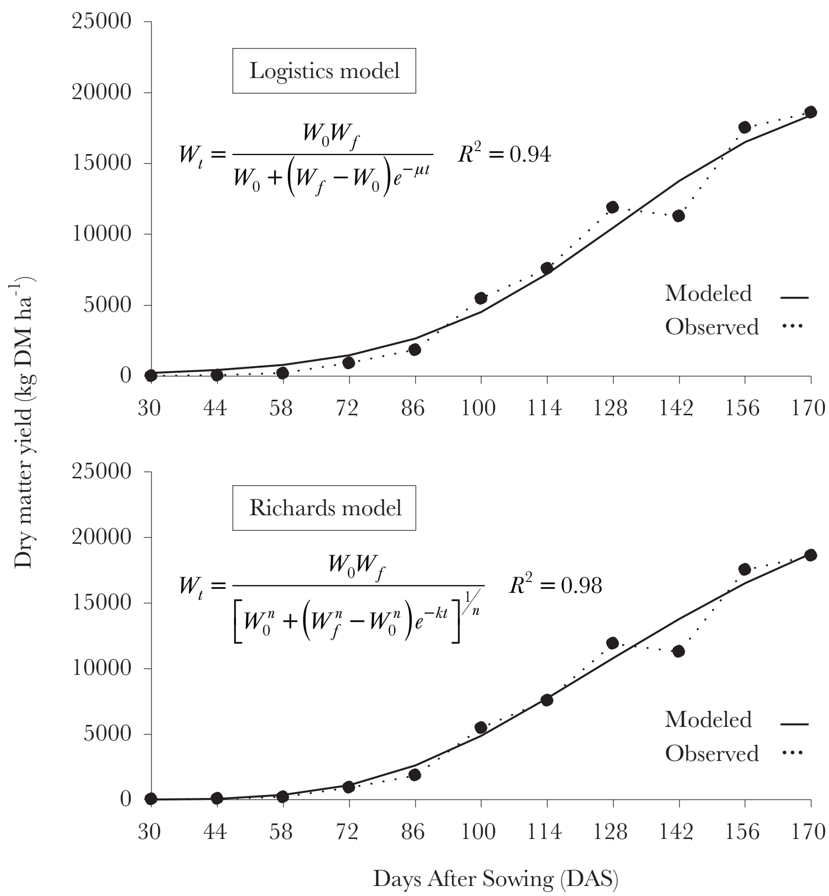

Figure 4. Dry matter yield (kg DM ha^{-1}) for A7573 maize hybrid obtained in the field and simulated with the Logistic and Richards models. Production cycle: April 8th to September 23rd, 2017. Location: southeastern Coahuila, Mexico.

overestimation was 8.5% (Logistic) and 13% (Richards). The lowest average yields were registered in the cultivar AN388, with 4488, 4585, and 4406 kg DM ha^{-1}, in the observed, Logistic, and Richards models, respectively. The Logistic model had an 2% overestimation, while the Richards model had an 1.8% underestimation. Paredes *et al.* (2014) mention that the prediction error is reduced when fewer differences are found between the observed values and those generated by the models, which indicates a correct parameterization of the forage yield curve.

CONCLUSIONS

The adjustment of the models accounted for 83% of the total variability of the dry matter yield. The Logistic model had the best fit for the AN447 and AN388 hybrids, while the Richards model had the best fit for the A7573. However, both models underestimated the observed performance. The A7573 and AN388 hybrids were the best and least fit, respectively.

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